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Research Article

An Advantageous Numerical Method for Solution of Linear Differential Equations by Stancu Polynomials

Neşe İşler Acar*

Faculty of Arts and Sciences, Department of Mathematics, Istiklal Campus, Burdur, 15030, Burdur Mehmet Akif Ersoy University, Turkey

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***Corresponding author:** Neşe İşler Acar, Faculty of Arts and Sciences, Department of Mathematics, Istiklal Campus, Burdur, 15030, Burdur Mehmet Akif Ersoy University, Turkey,
E-mail: nisler@mehmetakif.edu.tr, ni57662@gmail.com

ORCiD: https://orcid.org/0000-0003-3894-5950

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Abstract

In this study, a numerical method that is alternative to the Bernstein collocation method has been investigated for solution of the linear differential equations. The theory of the method has been constituted by considering the Stancu polynomials and their algebric properties. The applicability of the method has been indicated on initial and boundary value problems. In addition, the numerical results of the proposed method have been compared with the numerical results of the known method had the best approximation in the past studies. Therefore, whether usability and efficiency of the proposed method is or not has been presented.

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1. Introduction

Stancu [1] has produced the Stancu polynomials as a generalization of the Bernstein polynomials defined on the interval [0, 1] as follows:

$$S_n(y;x)=\sum_{i=0}^{n} y\left(\frac{i+\alpha}{n+\beta}\right)\binom{n}{i}x^i(1-x)^{n-i} \quad (1.1)$$

Where $0 \le x$ 1, $0 \le \alpha \le \beta$. The Stancu polynomials are the Bernstein polynomials for $\alpha = \beta = 0$, Therefore, these polynomials refered the Bernstein-Stancu polynomials by Altomare and Campiti [2]. Likewise, Stancu [3] indicated that the Stancu polynomials verified the Weierstrass Theorem [4] such as the Bernstein polynomials. In other words, the Stancu polynomials converge to a continuous function on the interval [0, 1]. Due to the fact that Stancu polynomials are Bernstein type polynomials, they have similar algebric properties with the Bernstein polynomials like positivity, continuity, recursion's relation, differentiability, integrability over the interval [0, 1]. Moreover, since the Stancu polynomials also depends on the parameters α and β, these polynomials can be used to obtain better approximation than the Bernstein polynomials at the points $x=\frac{\alpha}{\beta}$ on the interval [0, 1]. That is; a better approximation of a continuous function y at any x points on the interval [0, 1] can be effectively got to use the Stancu polynomials with relevant selections of parameters α and β than the Bernstein polynomials [5].

Since many physical sciences, biological sciences and engineering problems in chemical reaction kinetics, molecular dynamics, electronic circuits, population dynamics can be modelled by differential equations, numerical solutions of these equations have become very significant work items in many branches of science for a long time. Because polynomials have algebric properties like countunity, derivability and integrability, a lot of polynomials as Bernstein, Chebyshev,

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Legendre, Jacobi and Laguerre polynomials play very substantial role producing numerical methods for the solutions of the differential equations. The Bernstein polynomials are commonly used polynomials among the numerical methods procured by these polynomials. Only in the past decade, many studies on the numerical solutions of different types of differential equations have been concerned with the collocation method [6-11], Galerkin Method [12-14], operational matrix method [15-21], Adomian decomposition method [22,23] that were produced by considering the Bernstein polynomials. However any numerical method have not improved for numerical solutions of the differential and other equations by considering the Stancu polynomials. Up to now done studies are about approximation properties and convergence rate of the Stancu polynomials. For this reason, in this study, our aim is to investigate the numerical solutions of the linear differential equations

$$\sum_{k=0}^{m} a_k(x)y^{(k)}(x)=g(x), 0\le x\le 1 \tag{1.2}$$

under the initial conditions

$$\sum_{k=0}^{m-1}\lambda_{jk}y^{(k)}(c)=\mu_j;\ j=0,1,\ldots,m-1, c\in[0,1] \tag{1.3}$$

or boundary conditions

$$\sum_{k=0}^{m-1}\left[\alpha_{jk}y^{(k)}(a)+\beta_{jk}y^{(k)}(b)\right]=\gamma_j; j=0,1,\ldots,m-1, a,b\in[0,1] \tag{1.4}$$

by considering the Stancu polynomials (1.1) that are one of the generalization of the Bernstein polynomials. Here $a_k(x)$, $g(x)\in C[0,1]$ and $y(x)$ is unknown function. Moreover, $p_{i,n}(x)$ called the basis polynomials of the Bernstein type polynomials are mentioned in [24] and these polynomials have an important matrix relation that is underlined this study as follows:

Theorem 1.1. There is a relation between the basis polynomials matrix and their derivatives in the form

$$\mathbf{P}^{(k)}(x)=\mathbf{P}(x)\mathbf{N}^k; k=0,1,\ldots,m.$$

Here $\mathbf{P}(x)=\left[p_{i,n}(x)\right]$, $\mathbf{P}^{k}(x)=\left[p_{i,n}^{(k)}(x)\right]$ are $1\times(n+1)$ matrices, $N=(d_{ij})$ is $(n+1)\times(n+1)$ matrix such that the elements of N are defined by

$$d_{ij}=\begin{cases} n-i & ; \quad \text{if} j=i+1 \\ 2i-n & ; \quad \text{if} j=i \\ -i & ; \quad \text{if} j=i-1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

for $i, j = 0, 1, \ldots, n$ and $N^0 = I$ is identity matrix [25].

The paper's other sections are continued like that: In Section 2, the method depended on the Stancu polynomials and collocation points have been explained theoretically. In Section 3, some initial and boundary value problems have been considered to show how the method can be applied to the differential equations. Moreover, by getting the numerical results of the method's maximum and mean errors, the whether method is effective or not have been explored. In addition, the obtained numerical results have been compared with the numerical results of the other well known methods to see whether the method produced by Stancu polynomials is better than the other methods or not. In final Section 4, some inferences have been made about method's advantages and some advices that will lead to new studies have been given.

2. Manifestation the method

Theorem 2.1. Let $x_i=\dfrac{i+\alpha}{n+\beta}$ be collocation points on the interval [0, 1] and $y\in\mathbb{C}[0,1]$. By means of the Stancu polynomials' approach, the general m th-order linear differential equation (1.2) can expressed by a matrix equation as follows:

$$\sum_{k=0}^{m}\mathbf{A}_k\mathbf{P}\mathbf{N}^k\mathbf{Y}=\mathbf{G}. \tag{2.1}$$

Here the matrices are

$\mathbf{A}_k=diag\left[a_k(x_i)\right]$, $\mathbf{P}=\left[p_{j,n}(x_i)\right]$, $\mathbf{G}=\left[g(x_i)\right]$ and

$\mathbf{Y}=\left[y\left(\dfrac{i+\alpha}{n+\beta}\right)\right];\ i,j=0,\ldots,n$ such that $0 \le x \le 1$, $0 \le \alpha \le \beta$.

Proof: By considering Theorem 1.1 and the Stancu polynomials (1.1), unknown functions and their derivatives can be expressed by matrix form as follows:

$$y^{(k)}(x)\simeq S_n^{(k)}(y;x)=\mathbf{P}(x)\mathbf{N}^k\mathbf{Y}; k=0,1,\ldots,m. \tag{2.2}$$

Let $x_i=\dfrac{i+\alpha}{n+\beta}$ be collocation points that are nodes of the Stancu polynomials. These points are depended on selections of the α and β values. Substituting the collocation points and matrix equation (2.2) into equation (1.2), the following algebraic equation system is obtained

$$\sum_{k=0}^{m}a_k(x_i)\mathbf{P}(x_i)\mathbf{N}^k\mathbf{Y}=g(x_i); i=0,\ldots,n$$

such that $y^{(k)}(x_i)=S_n^{(k)}(y;x_i);\ k=0,\ldots,m$. Here the matrices are defined as follows:

$$\mathbf{P}=\begin{bmatrix}\mathbf{P}(x_0)\\ \mathbf{P}(x_1)\\ \vdots\\ \mathbf{P}(x_n)\end{bmatrix}=\begin{bmatrix}p_{0,n}(x_0) & p_{1,n}(x_0) & \cdots & p_{n,n}(x_0)\\ p_{0,n}(x_1) & p_{1,n}(x_1) & \cdots & p_{n,n}(x_1)\\ \vdots & & \ddots & \vdots\\ p_{0,n}(x_n) & p_{1,n}(x_n) & & p_{n,n}(x_n)\end{bmatrix}, \mathbf{Y}=\begin{bmatrix}y(x_0)\\ y(x_1)\\ \vdots\\ y(x_n)\end{bmatrix},$$

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$$\mathbf{A}_k=\begin{bmatrix} a_k(x_0) & 0 & \cdots & 0 \\ 0 & a_k(x_1) & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & a_k(x_n) \end{bmatrix},\mathbf{G}=\begin{bmatrix} g(x_0) \\ g(x_1) \\ \vdots \\ g(x_n) \end{bmatrix}.$$

The proof is completed thanks to this equation system.

Some steps can be followed order to solve the differential equations (1.2) under the initial (1.3) and boundary (1.4) conditions.

Step 1. The equation (2.1) can be written compactly as follows:

$$\mathbf{WY}=\mathbf{G}\text{or}\left[\mathbf{W;G}\right];\ \mathbf{W}=\sum_{k=0}^{m}\mathbf{A}_k\mathbf{PN}^k. \qquad (2.3)$$

This matrix equation (2.3) indicates a linear algebraic system including unknown coefficients $y_0, y_1, .., y_n$.

Step 2. Initial (1.2) and boundary (1.3) conditions are also stated by matrix equations as follows:

$$\mathbf{I}_j=\left[\imath_{j,k}\right]=\sum_{k=0}^{m-1}\lambda_{jk}\mathbf{P}(c)\mathbf{N}^k,\ \mathbf{I}_j\mathbf{Y}=\mu_j\text{or}\left[\mathbf{I}_j;\mu_j\right], \qquad (2.4)$$

$$\mathbf{U}_j=\left[u_{j,k}\right]=\sum_{k=0}^{m-1}\left[\alpha_{jk}\mathbf{P}(a)\mathbf{N}^k+\beta_{jk}\mathbf{P}(b)\mathbf{N}^k\right],\ \mathbf{U}_j\mathbf{Y}=\gamma_j\text{or}\left[\mathbf{U}_j;\gamma_j\right]. \qquad (2.5)$$

Step 3. Adding or deleting techniques can be used in order to get the solution of equation (1.2) under the initial (1.3) and boundary (1.4) conditions. In adding technique the elements of the row matrices (2.4) or (2.5) are added to the end of the matrix (2.3). By this way, an augmented matrix is obtained as follows:

$$\left[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}\right]=\begin{bmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} & ; & g(x_0) \\ \cdots & \cdots & \cdots & \cdots & ; & \cdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} & ; & g(x_n) \\ \imath_{0,0} & \imath_{0,1} & \cdots & \imath_{0,n} & ; & \mu_0 \\ \cdots & \cdots & \cdots & \cdots & ; & \cdots \\ \imath_{m-1,0} & \imath_{m-1,1} & \cdots & \imath_{m-1,n} & ; & \mu_{m-1} \end{bmatrix}.$$

This matrix is also a $(n+m+1)\times(n+1)$ dimensional matrix. In replacing technique, rows of the augmented matrix (2.3) are replaced with the rows of the matrices (2.4) or (2.5). For example, replacing the last row matrices with the initial conditions, the new obtained matrix is a square matrix as below

$$\left[\mathbf{W}^*;\mathbf{G}^*\right]=\begin{bmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} & ; & g(x_0) \\ \cdots & \cdots & \cdots & \cdots & ; & \cdots \\ w_{n-m,0} & w_{n-m,1} & \cdots & w_{n-m,n} & ; & g(x_{n-m}) \\ \imath_{0,0} & \imath_{0,1} & \cdots & \imath_{0,n} & ; & \mu_0 \\ \cdots & \cdots & \cdots & \cdots & ; & \cdots \\ \imath_{m-1,0} & \imath_{m-1,1} & \cdots & \imath_{m-1,n} & ; & \mu_{m-1} \end{bmatrix}.$$

Step 4. If $rank(\widetilde{\mathbf{W}})=rank\left[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}\right]=n+1$, the system can be written as $\mathbf{Y}=\left(\widetilde{\mathbf{W}}\right)^{-1}\mathbf{G}$ or $\mathbf{Y}=\left(\mathbf{W}^*\right)^{-1}\mathbf{G}$ and the unknown coefficients $y_{i;\ i=0,1,\dots,n}$ of the system are uniquely specified.

3. Some applications of the method

In this section, some initial and boundary problems have been considered in order to show that the Stancu collocation method can be applied to the problems. The numerical results have been given on different collocation points in according to values of α and β. In addition, the numerical results have been calculated on the programme MATLAB 7.1. Moreover, the numerical results have been presented as the tables. Likewise, to see how much the Stancu collocation method is useful and effective, the results of maximum and mean errors have been compared with the results of the other methods. Let define the used errors as follows:

Definition 3.1. Let $y(x)$ is an exact solution and $S_n(y;x)$ is a Stancu approximate solution. Maximum and mean errors can be determined on the collocation points by the following relations.

$$E_{\max}=\max_{x_i}\left|y(x_i)-S_n(y;x_i)\right|,\ E_{mean}=\frac{1}{n}\sum_{i=1}^{n}\left|y(x_i)-S_n(y;x_i)\right|.$$

Example 3.1. Let the following initial value problem [25] consider:

$$y''+xy'-2y=x\cos x-3\sin x;\ 0\le x\le 1$$

$$y(0)=0,\ y'(0)=1.$$

The exact solution of this problem is $y(x)=\sin x$.

In Table 1, the mean errors of the proposed method have been given for the different values of α = β and α < β. The numerical results of the method also have been calculated with adding technique. Moreover, the numerical results of the Stancu collocation method have been compared with the numerical results of the Bernstein collocation method for different values of n. The best numerical results of the proposed method have been obtained for α = 0 and β = 2. Furthermore, the table shows that the numerical results of the proposed method are better than the numerical results of the Berstein collation method for each value of n presented by Akyüz-Daşcıoğlu and Acar İşler [25].

Example 3.2. Let the following boundary value problem [26] consider:

Table 1: Comparison of the E_{mean} errors for Example 3.1.

	Stancu Collocation Method					Bernstein Collocation Method [25]
n	$\alpha=0.25$ $\beta=0.25$	$\alpha=2.5\times10^{-4}$ $\beta=2.5\times10^{-4}$	$\alpha=2.5\times10^{-3}$ $\beta=7.5\times10^{-3}$	$\alpha=0$ $\beta=1$	$\alpha=0$ $\beta=2$	$\alpha=0$ $\beta=0$
5	6.3e − 002	7.8e − 005	6.9e − 004	1.0e − 005	9.0e − 007	9.6e − 006
10	3.2e − 002	3.4e − 005	3.4e − 004	5.1e − 013	2.0e − 013	1.4e − 012
15	2.2e − 002	2.2e − 005	2.2e − 004	3.3e − 015	8.9e − 016	1.3e − 015
20	1.6e − 002	1.7e − 005	1.7e − 004	5.4e − 015	8.7e − 016	2.6e − 015

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$$y^{(4)} + y^{(3)} = x - 1; 0 \le x \le 1$$

$$y(0) = 1, y'(0) = -1, y''(0) = 2, y'''(0) = -2.$$

that the exact solution of the problem is $y(x) = 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}$.

In Table 2, the maximum errors of the proposed method have been compared with the maximum errors of the Berstein collocation method for different values of α and β. Both numerical results of the proposed method and Bernstein collocation method have been obtained by using replacing technique. In replacing technique, last rows of the augmented matrix have been deleted and row marices (2.4) have been written instead of it. In respect of Table 2, the numerical results of the proposed method are better than the numerical results of the Bernstein collocation method given in doctoral thesis by İşler Acar [26] for different values of n. This mean that numerical results calculated on the points $x_i = \frac{i}{n+2}$; $n = 0,1,\ldots n$ are more effective than the numerical results calculated on the points $x_i = \frac{i}{n}$; $n = 0,1,\ldots n$. Moreover, when the exact solution is polynomial function, the numerical results are effectively good for n values run around the degree of polynomial.

Example 3.3. Let the following the initial value problem [27] that has exact solution $y(x) = (1-x)e^x$ consider:

$$y^{(8)} - y = -8e^x; 0 < x < 1$$

$$y(0) = 1, y'(0) = 0, y^{(2)}(0) = -2, y^{(3)}(0) = -2, y^{(4)}(0) = -3$$

$$y^{(5)}(0) = -4, y^{(6)}(0) = -5, y^{(7)}(0) = -6.$$

In Table 3, the maximum errors of the proposed method have been given on the different collocation points depended on the values of α and β. Moreover, the numerical results of the Stancu collocation method have been comparised with the numerical results of the Bernstein collocation method. By the Table 3, the numerical results of the proposed method are better than the other method's numerical results. Since the differential equation is not define on the 0 and 1 points, the numerical results computed on the collocation points $x_i = \frac{i}{n+2}$ are really more effective and the method converges faster than the numerical results computed on the collocation points $x_i = \frac{i}{n}$. Moreover, the numerical results of the proposed method have been computed with adding technique. Generally, Table 3 shows that the numerical results of the proposed method recover quickly for values of α < β.

Example 3.4. Let the following the boundary value problem [15] that has exact solution $y(x)=e^x$ consider:

$$y^{(4)} - 3y = -2e^x; x \in 0,1]$$

$$y(0) = 1, y(1) = e, y'(0) = 1, y'(1) = e.$$

In Table 4, the Stancu collocation method's maximum errors have been compared with the other methods' maximum errors. By the Table 4, the Stancu collocation method converges faster for values α = 0 and β = 2 than the Berstein collocation method for increasing n values. This means that the numerical results computed on the collocation points $x_i = \frac{i}{n+2}$ are effectively better than the numerical results computed on the collocation points $x_i = \frac{i}{n}$. Besides, the numerical results of the Stancu collocation method are noticeably better than the numerical results of the Sin-Galerkin Method and Modified Decomposition Method [28] for n = 14. On the other hand the numerical results of the Stancu collocation method get better slowly than the BPG Method's numerical results [15] for increasing n values.

Table 2: Comparison of the E_{max} errors for Example 3.2.

	Stancu Collocation Method				Bernstein Collocation Method [26]
n	$\alpha = 0.5$ $\beta = 0.5$	$\alpha = 5.0 \times 10^{-4}$ $\beta = 5.0 \times 10^{-4}$	$\alpha = 2.5 \times 10^{-3}$ $\beta = 7.5 \times 10^{-3}$	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 0$
3	6.1e – 001	1.2e – 004	3.1e – 016	1.9e – 016	1.1e – 016
4	5.4e – 001	5.6e – 005	2.2e – 016	3.1e – 016	0
5	4.5e – 001	2.8e – 005	4.4e – 016	4.4e –016	4.4e – 016
6	1.6e – 001	2.0e – 005	8.9e – 016	0	6.7e – 016

Table 3: Comparison of the E_{max} errors for Example 3.2.

	Stancu Collocation Method					Bernstein Collocation Method
n	$\alpha = 0.01$ $\beta = 0.01$	$\alpha = 5.0 \times 10^{-4}$ $\beta = 5.0 \times 10^{-4}$	$\alpha = 7.5 \times 10^{-5}$ $\beta = 1.0 \times 10^{-5}$	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 2$	$\alpha = 0$ $\beta = 0$
10	3.3e – 003	7.1e – 004	5.9e – 004	2.3e – 004	9.7e – 005	5.7e – 004
15	1.8e – 003	9.1e – 005	1.4e – 004	2.2e – 012	5.5e – 012	9.0e – 012
20	1.4e – 003	6.8e – 005	1.0e – 005	3.3e – 011	7.5e – 012	2.4e – 010
25	1.1e – 003	5.4e – 005	8.1e – 006	2.3e – 010	8.2e – 010	2.2e – 009
30	9.1e – 004	4.5e – 005	6.8e – 006	6.0e – 009	1.2e – 009	8.4e – 009

Table 4: Comparison of the E_{max} errors for Example 3.4.

	Stancu Collocation Method		Bernstein Collocation Method	Bernstein-Petrov-Galerkin Method [15]	Sin-Galerkin Method [28]	Modified Decomposition Method [28]
n	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 2$	$\alpha = 0$ $\beta = 0$			
8	1.3e - 007	6.5e - 008	2.8e - 007	7.3e - 010		
10	1.8e - 010	8.8e - 011	4.1e - 010	1.8e - 013		
12	6.9e - 014	4.2e - 013	1.1e - 012	4.5e - 015		
14	2.2e - 013	1.6e - 015	4.5e - 015	1.3e - 015	3.7e - 009	2.5e - 008

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4. Discussion and implications

In this study, a numerical method has been produced in terms of the Stancu polynomials that are generalization of the Bernstein polynomials for the solution of linear differential equations under the initial and boundary conditions on the $C[0, 1]$. The method bases on the collocation method. The collocation points of the method depend on the selection of the α and β values. Adding and replacing techniques have been used in order to compute the numerical results. The numerical results have been made in programme MATLAB. Since the Matlab is a high-performance software written mainly for technical and scientific calculations, including numerical calculation, graphical data representation and programming, this programme has been preferred to compute the numerical results. Moreover, the numerical results of the method have been compared with the numerical results of the other methods. Since the Bernstein collocation method is far effective method and the Stancu polynomials are one of the Bernstein type polynomials, the numerical results of the method generally compared with the numerical results of the Bernstein collocation method. Likewise, the numerical results have been given on the Tables. By considering this study, we can a lot of positive inferences as follows:

i. The Stancu collocation method is easy understandable and applicable to the problems like the Bernstein collocation method.

ii. By using the Stancu collocation method, a better approximation of a continuous unknown function y at any collocation points x on the interval $[0, 1]$ can be effectively got with relevant selection of the parameters α and β than the Bernstein collocation method. This state also provides to limit calculation to less number of terms.

iii. The numerical results of the Stancu collocation method get better for values of $\alpha = 0$ and $\beta = 2$ than the other methods for increasing n values.

iv. To find the best numerical results of the method, both adding and replacing techniques can be used.

v. To write the codes of the method and compute the numerical results of the method in MATLAB programme is very easy and clear because of the MATLAB has very large library.

In the direction of all these inferences, the Stancu polynomials that is more general structure of the Bernstein polynomials can be used for numerical solution of the initial and boundary value problems at the computational methods. Thanks to this study, new alternative collocation methods based on the Bernstein type polynomials such as Bernstein-Cholodowsky polynomials [29], Stancu-Cholodowsky polynomials [30] as can be investigated in the future studies. Moreover, new improved numerical methods can be tasted on the numerical solutions of many types of linear and nonlinear differential equations like integro-differential equations, partial differential equations, fractional differential equations modeling the natural and engineering problems. The applications of the Stancu collocation method and new obtained methods can also be coded with different programmes like Mapple, Mathematica, Matcad except MATLAB.

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